

Propagation and Absorption of Electromagnetic Waves in a Coaxial Plasma-Waveguide-System ($m=1$)

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The dispersion relation of electromagnetic waves propagating in a coaxial plasma-waveguide-system is solved for a cold plasma. The influence of losses, discharge tube, and radial gradients of electron density are discussed in detail. Strong absorption is found near the dipole resonance of the plasma column.

1. Introduction

In a nonmagnetized homogeneous plasma an electromagnetic wave of a frequency ω propagates when $\omega > \omega_p$, where ω_p is the plasma frequency (i.e. when the electron density is below the cut-off density: $n < n_c$). For $\omega < \omega_p$ the plasma is opaque and no wave propagation is possible; the amplitude of the wave decreases exponentially inside the plasma and the wave is reflected almost totally at the boundary of the plasma (Heald and Wharton¹).

In case of a special geometry, the high reflectivity of an overdense plasma may be used to get strong absorption of electromagnetic waves by means of multiple reflections (Beerwald et al.²). Moreover, a homogeneous plasma column exhibits a dipole resonance at $\omega = \omega_p/\sqrt{2}$ — resp. $n = 2 n_c$ (Tonks^{3,4}) — which has been used in a coaxial plasma-waveguide-system to get strong absorption of an electromagnetic wave in case of an overdense plasma column by Beerwald and Kampmann⁵.

The propagation of microwaves in waveguides loaded with plasma has been treated — mostly in the electrostatic limit — by many authors for $\omega < \omega_p/\sqrt{2}$; a survey is given e.g. in Reference^{6,7}. The electromagnetic dispersion relation has been studied for a homogeneous lossless plasma column in a circular waveguide by Gehre et al.⁸ for $\omega \leq \omega_p$. For comparison with experimental work inclusion of losses as small perturbation is not sufficient. For quantitative comparisons and for considerations of power transfer complete inclusion of losses is mandatory. This work undertakes to carry out these calculations in full electromagnetic treatment. Furthermore density gradients are taken into

account which have great influence on the absorption in case of high electron densities. The method used to calculate the influence of the density gradients reduces to the approach outlined by Kinderdijk and Hagebeuk⁹ for low electron densities.

The azimuthal $m=1$ -symmetry of the fields has been chosen for the calculations because the H_{11} -waveguide mode represents the fundamental mode of the empty circular waveguide. The use of the fundamental mode provides an advantage for experimental investigations since interference of different propagating modes can be avoided in case of a sufficiently low frequency ω .

2. The Solution of the Dispersion Relation

The dispersion relation is solved for a plasma-waveguide-system using the following assumptions:

- i) The waveguide has circular cross section. The metallic wall has infinite conductivity. The length of the plasma-waveguide-system (“z-direction”) is infinite.
- ii) Cylindrical stratified dielectrics are situated coaxially inside the waveguide. The refractive index of the dielectrics has to be complex in order to take into account absorption.
- iii) The azimuthal symmetry of the fields is described by $m=1$.
- iv) Charge density $\rho=0$. Macroscopic current density $\mathbf{j}=0$; $\mu=1$ for the plasma and for the discharge tube.

The Maxwell equations are noted in SI units; according to iv) they are written:

$$\vec{\nabla} \times \mathbf{E} + \mu_0 (\partial \mathbf{H} / \partial t) = 0, \quad (1)$$

$$\vec{\nabla} \cdot (\epsilon \cdot \mathbf{E}) = 0, \quad (2)$$

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$$\vec{\nabla} \times \mathbf{H} - \varepsilon \varepsilon_0 (\partial \mathbf{E} / \partial t) = 0, \quad (3)$$

$$\vec{\nabla} \cdot (\mu_0 \cdot \mathbf{H}) = 0. \quad (4)$$

When the relative dielectric constant ε is nonuniform (i. e. $\vec{\nabla} \varepsilon \neq 0$) the wave equation for the magnetic field is given by:

$$\Delta \mathbf{H} - \frac{\varepsilon}{c^2} \cdot \frac{\partial^2}{\partial t^2} \mathbf{H} + \frac{\vec{\nabla} \varepsilon}{\varepsilon} \times (\vec{\nabla} \times \mathbf{H}) = 0 \quad (5)$$

and for the electric field:

$$\Delta \mathbf{E} - \frac{\varepsilon}{c^2} \cdot \frac{\partial^2}{\partial t^2} \mathbf{E} + \vec{\nabla} \left(\frac{\vec{\nabla} \varepsilon}{\varepsilon} \cdot \mathbf{E} \right) = 0. \quad (6)$$

The magnetic and electric fields are described in cylinder coordinates for linear polarization of the wave:

$$\mathbf{H}(r, \varphi, z) = \mathbf{H}(r) \cdot \cos \varphi \cdot e^{i(\omega t - kz)} \quad (7)$$

$$\mathbf{E}(r, \varphi, z) = \mathbf{E}(r) \cdot \sin \varphi \cdot e^{i(\omega t - kz)}, \quad (8)$$

here φ means the azimuth angle and k represents the wave vector in z -direction, i. e. in the direction of propagation of the wave.

Using Eqs. (1) – (4), the r - and φ -components of \mathbf{H} and \mathbf{E} can easily be expressed in terms of $H_z(r)$ and $E_z(r)$:

$$H_r = -i \frac{k}{b^2} \frac{\partial H_z}{\partial r} + i \frac{\varepsilon \varepsilon_0 \omega}{b^2 \cdot r} \cdot E_z, \quad (9)$$

$$H_\varphi = i \frac{k}{b^2 \cdot r} \cdot H_z - i \frac{\varepsilon \varepsilon_0 \omega}{b^2} \cdot \frac{\partial E_z}{\partial r}, \quad (10)$$

$$E_r = i \frac{\mu_0 \omega}{b^2 \cdot r} \cdot H_z - i \frac{k}{b^2} \cdot \frac{\partial E_z}{\partial r}, \quad (11)$$

$$E_\varphi = i \frac{\mu_0 \omega}{b^2} \cdot \frac{\partial H_z}{\partial r} - i \frac{k}{b^2 \cdot r} \cdot E_z \quad (12)$$

where

$$b^2 = \frac{\omega^2}{c^2} \varepsilon - k^2. \quad (13)$$

In the special case when ε shows only radial dependence, the wave equations for H_z and E_z reduce to:

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} H_z \right) + \left(b^2 - \frac{1}{r^2} \right) H_z = \frac{\omega^2}{b^2 c^2} \frac{\partial \varepsilon}{\partial r} \left(\frac{\partial}{\partial r} H_z - \frac{k}{\mu_0 \omega r} E_z \right) \quad (14)$$

and

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} E_z \right) + \left(b^2 - \frac{1}{r^2} \right) E_z = \frac{k^2}{b^2 \cdot \varepsilon} \frac{\partial \varepsilon}{\partial r} \left(\frac{\partial}{\partial r} E_z - \frac{\mu_0 \omega}{k \cdot r} H_z \right). \quad (15)$$

When $\varepsilon(r)$ is given, one has to solve two coupled differential equations of second order [(14) and (15)]: for given frequency ω one has to calculate a wave vector k so that the tangential component of the electric field will be zero at the metallic boundary of the waveguide ($r = a$):

$$E_\varphi(a) = 0, \quad E_z(a) = 0. \quad (16)$$

For a homogeneous dielectric the right hand side of (14) and (15) is zero. In this case the differential equations are not coupled and one gets the H_{1n} - and E_{1n} -modes as solutions of the homogeneous waveguide; the lowest modes are H_{11} and E_{11} .

When two concentric homogeneous dielectrics are situated inside the waveguide, the differential equations are coupled only at the interface between the dielectrics, but it is not possible to fulfill the conditions of continuity at the interface and the boundary conditions at the wall (16) e. g. with a H_{11} -mode; the solution of the wave equation has components as well of H_z as of E_z and therefore is called a “hybrid mode”. Only at the cut-off the solution may be identified as transversal mode.

The analytic solution of (14) and (15) is possible only for special functions of $\varepsilon(r)$ (e. g. Ref. ¹⁰; a survey is given by Wharton ¹¹, where also questions of nomenclature are discussed). — Since numerical integration of (14) and (15) is problematic, a method is outlined to solve the dispersion relation in case of a step profile of dielectrics; this allows to take into account a discharge tube and even steep gradients of electron density.

The waveguide is divided into s coaxial homogeneous layers with ε_i the relative dielectric constant and a_i the outer radius of the i -th layer, as shown in Figure 1.

Since $\varepsilon(r)$ is constant within each layer, the wave equations (14) and (15) reduce inside each layer

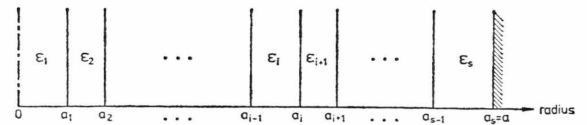


Fig. 1. Distribution of the layers along the radius of the waveguide.

to

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} H_z \right) + \left(b^2 - \frac{1}{r^2} \right) H_z = 0, \quad (17)$$

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} E_z \right) + \left(b^2 - \frac{1}{r^2} \right) E_z = 0. \quad (18)$$

The solution of (17) and (18) are the Bessel and Neumann functions of first order. So we get inside the i -th layer:

$$H_z = A_i J_1(b_i r) + B_i Y_1(b_i r), \quad (19)$$

$$E_z = C_i J_1(b_i r) + D_i Y_1(b_i r) \quad (20)$$

where corresponding to (13): $b_i^2 = \varepsilon_i \omega^2 / c^2 - k^2$ (21).

At each interface the solutions of two layers are coupled by the conditions that H_φ , H_z , E_φ and E_z must be continuous — the continuity of B_r and D_r is then given implicitly as one can proof easily. At the interface of the layers i and $i+1$ the continuity of the fields may be written with (9) – (13) and (19), (20) as:

$$H_\varphi: \quad \frac{i k}{b_i^2 a_i} [A_i J_1(b_i a_i) + B_i Y_1(b_i a_i)] \quad (22)$$

$$\begin{aligned} & -i \frac{\varepsilon \varepsilon_0 \omega}{b_i} [C_i J_1'(b_i a_i) + D_i Y_1'(b_i a_i)] \\ & = -\frac{i k}{b_{i+1}^2 a_i} [A_{i+1} J_1(b_{i+1} a_i) + B_{i+1} Y_1(b_{i+1} a_i)] \\ & -i \frac{\varepsilon \varepsilon_0 \omega}{b_{i+1}} [C_{i+1} J_1'(b_{i+1} a_i) + D_{i+1} Y_1'(b_{i+1} a_i)]. \end{aligned}$$

$$H_z: \quad A_i J_1(b_i a_i) + B_i Y_1(b_i a_i) = A_{i+1} J_1(b_{i+1} a_i) + B_{i+1} Y_1(b_{i+1} a_i). \quad (23)$$

$$E_\varphi: \quad \frac{i \mu_0 \omega}{b_i} [A_i J_1'(b_i a_i) + B_i Y_1'(b_i a_i)] \quad (24)$$

$$\begin{aligned} & -\frac{i k}{b_i^2 a_i} [C_i J_1(b_i a_i) + D_i Y_1(b_i a_i)] \\ & = \frac{i \mu_0 \omega}{b_{i+1}} [A_{i+1} J_1'(b_{i+1} a_i) + B_{i+1} Y_1'(b_{i+1} a_i)] \\ & -\frac{i k}{b_{i+1}^2 a_i} [C_{i+1} J_1(b_{i+1} a_i) + D_{i+1} Y_1(b_{i+1} a_i)]. \end{aligned}$$

$$E_z: \quad C_i J_1(b_i a_i) + D_i Y_1(b_i a_i) = C_{i+1} J_1(b_{i+1} a_i) + D_{i+1} Y_1(b_{i+1} a_i). \quad (25)$$

The prime denotes differentiation with respect to the argument of the Bessel and Neumann functions.

When the solution is known in the i -th layer, the four equations (22) – (25) will give the four unknown coefficients of the solution in the $(i+1)$ -th layer:

$$A_{i+1} = \frac{A_i J_1(b_i a_i) + B_i Y_1(b_i a_i) - B_{i+1} Y_1(b_{i+1} a_i)}{J_1(b_{i+1} a_i)}, \quad (26)$$

$$\begin{aligned} B_{i+1} = & \frac{1}{J_1(b_{i+1} a_i) Y_1'(b_{i+1} a_i) - J_1'(b_{i+1} a_i) Y_1(b_{i+1} a_i)} \cdot \\ & \left\{ \frac{k J_1(b_{i+1} a_i)}{\mu_0 \omega b_{i+1} a_i} \cdot \left\{ 1 - \left[\frac{b_{i+1}}{b_i} \right]^2 \right\} \right. \\ & \cdot [C_i J_1(b_i a_i) + D_i Y_1(b_i a_i)] \\ & + \frac{b_{i+1}}{b_i} J_1(b_{i+1} a_i) [A_i J_1'(b_i a_i) + B_i Y_1'(b_i a_i)] \\ & \left. - J_1'(b_{i+1} a_i) [A_i J_1(b_i a_i) + B_i Y_1(b_i a_i)] \right\}, \quad (27) \end{aligned}$$

$$C_{i+1} = \frac{C_i J_1(b_i a_i) + D_i Y_1(b_i a_i) - D_{i+1} Y_1(b_{i+1} a_i)}{J_1(b_{i+1} a_i)}, \quad (28)$$

$$\begin{aligned} D_{i+1} = & \frac{1}{J_1(b_{i+1} a_i) Y_1'(b_{i+1} a_i) - J_1'(b_{i+1} a_i) Y_1(b_{i+1} a_i)} \cdot \\ & \left\{ \frac{k J_1(b_{i+1} a_i)}{\varepsilon_{i+1} \varepsilon_0 \omega b_{i+1} a_i} \cdot \left\{ 1 - \left[\frac{b_{i+1}}{b_i} \right]^2 \right\} \right. \\ & \cdot [A_i J_1(b_i a_i) + B_i Y_1(b_i a_i)] \\ & + \frac{b_{i+1} \varepsilon_i}{b_i \varepsilon_{i+1}} J_1(b_{i+1} a_i) [C_i J_1'(b_i a_i) + D_i Y_1'(b_i a_i)] \\ & \left. - J_1'(b_{i+1} a_i) [C_i J_1(b_i a_i) + D_i Y_1(b_i a_i)] \right\}. \quad (29) \end{aligned}$$

In this notation, B_{i+1} and D_{i+1} must be calculated before being inserted into (26) resp. (28).

Since the solution has to be finite on the axis, the coefficients of the Neumann function have to be zero inside the first layer:

$$B_1 = D_1 = 0. \quad (30)$$

As the amplitude of the solution may be chosen arbitrarily, we choose $A_1 = 1$ (resp. $A_1 = (1; 0)$ in case of damping).

The boundary conditions (16) at the metallic wall may now be written — using (19), (20) and (12):

$$\begin{aligned} C_s J_1(b_s a_s) + D_s Y_1(b_s a_s) &= 0, \\ A_s J_1'(b_s a_s) + B_s Y_1'(b_s a_s) &= 0. \end{aligned} \quad (31)$$

Provided that ω and a set of ε_i and a_i are given, solving the dispersion relation means to find coefficients ($k; C_1$) that — considering the continuity conditions at the interfaces (26) — (29) the boundary conditions (31) are fulfilled.

For a given value of ($k; C_1$) one calculates the coefficients ($A_i; B_i; C_i; D_i$) with increasing i and then proves (31). In case (31) is not fulfilled, the value of ($k; C_1$) may be corrected iteratively by Newton's method (the derivatives may be substituted by difference quotients; e. g. Bellmann¹²). The iteration is stopped, when the tangential electric field strength at the metallic wall is small enough, e. g.

$$|E_\varphi(a)| + |E_z(a)| < 10^{-5} \cdot \text{Max}(|E|) \quad (32)$$

where $\text{Max}(|E|)$ denotes the maximum value of $|E|$ over the radius.

When dielectrics and plasmas with losses shall be treated, one has to use complex dielectric constants ε_i and then gets a complex wave vector k . For a cold plasma we use — according e. g. to Ref.¹:

$$\varepsilon_p = 1 - \frac{\omega_p^2}{\omega^2 \cdot [1 - i(\nu/\omega)]} = 1 - \frac{n}{n_c \cdot [1 - i(\nu/\omega)]} \quad (33)$$

where ν is the collision frequency.

As k and C_1 are complex, the iteration will take place in a four dimensional space. In order to get the power carried by the waveguide for a given field strength we may use Poynting's theorem:

$$P_z = \frac{1}{2} \int_0^a \int_0^{2\pi} (\mathbf{E} \times \mathbf{H}^*) \cdot \mathbf{e}_z r d\varphi dr$$

$$= \frac{1}{2} \int_0^a \int_0^{2\pi} (E_r \cdot H_\varphi^* \sin^2 \varphi - E_\varphi \cdot H_r^* \cos^2 \varphi) r d\varphi dr$$

$$= \frac{1}{2} \pi \int_0^a (E_r \cdot H_\varphi^* - E_\varphi \cdot H_r^*) r dr; \quad (34)$$

here \mathbf{e}_z is the unity vector in z -direction. The integration is carried out numerically.

A FORTRAN IV programm has been written to solve the dispersion relation according to the methods outlined above. For special parameters:

1. no damping, 2 concentric dielectric layers inside the waveguide (Chang and Dawson¹³)
2. no damping, discharge tube and plasma with parabolic profile of electron density inside the waveguide, $n < 0.25 \cdot n_c$ (Kinderdijk and Hagebeuk⁹)

the results have been compared to the results of other authors. Within the accuracy of diagrams no difference could be found.

3. Numerical Results

The results of the calculations — for typical parameters — are given in a way that facilitates the comparison with experiments: the normalized wave vector $k \cdot a$ is given as a function of the relative electron n/n_c . The frequency ω is constant and is referred to by $\omega a/c$ where $\omega/c = k_0$ is the wave vector for free propagation in vacuum; $\omega a/c$ compares the wave vector in vacuum with the radius of the waveguide whereas ka compares the wave vector in the plasma-waveguide-system with the radius of the waveguide.

As mentioned before, ω is chosen above the cut-off frequency of the empty waveguide; so propagation of electromagnetic waves is possible as well in

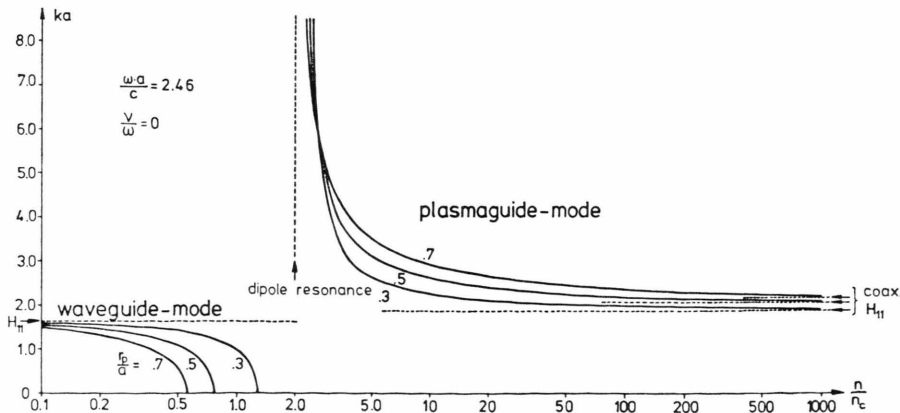


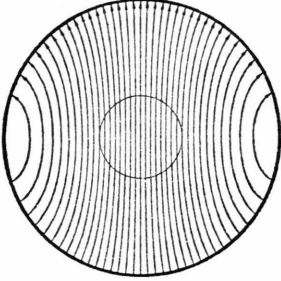
Fig. 2. Dispersion relation for collisionless homogeneous plasma columns in a circular waveguide.

empty
waveguide

$$\frac{\omega \cdot a}{c} = 2.46$$

$$\frac{r_p}{a} = 0.3$$

$$\frac{\nu}{\omega} = 0$$



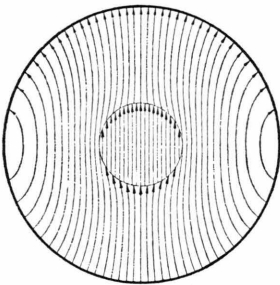
$$\frac{n}{n_c} = 0$$

$$ka = 1.631$$

$$d = 1$$

Fig. 3. Field patterns of the electric field strength in the cross section of a plasma-waveguide-system. (The electric field strength has been reduced by a scaling factor d .)

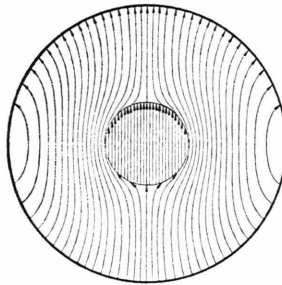
waveguide-mode



$$\frac{n}{n_c} = 0.4$$

$$ka = 1.473$$

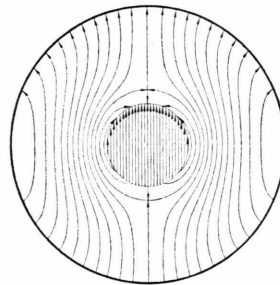
$$d = 1$$



$$\frac{n}{n_c} = 0.8$$

$$ka = 1.208$$

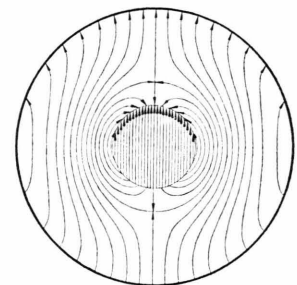
$$d = 1$$



$$\frac{n}{n_c} = 1.2$$

$$ka = 0.535$$

$$d = 0.5$$

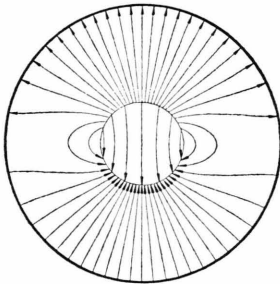


$$\frac{n}{n_c} = 1.4$$

$$ka = -0.859 i$$

$$d = 0.5$$

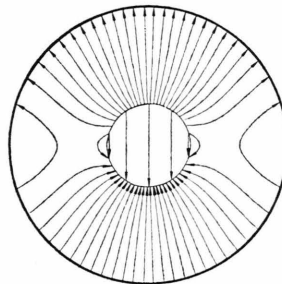
plasmaguide-mode



$$\frac{n}{n_c} = 2.7$$

$$ka = 5.942$$

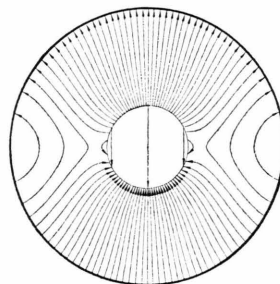
$$d = 0.1$$



$$\frac{n}{n_c} = 5.0$$

$$ka = 2.645$$

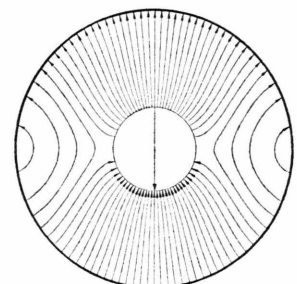
$$d = 0.5$$



$$\frac{n}{n_c} = 10$$

$$ka = 2.272$$

$$d = 1$$



$$\frac{n}{n_c} = 50$$

$$ka = 2.024$$

$$d = 1$$

the empty waveguide as for very high electron densities.

3.1. Homogeneous Plasma Column in a Circular Waveguide – without Collisions ($\nu = 0$)

At first the most simple case is discussed to show the peculiarity of the wave propagation in a plasma-waveguide-system (cf. Reference⁸). Figure 2 depicts the dispersion relation for several values of r_p/a , where r_p is the radius of the plasma column. The dispersion relation is divided into two regions:

- i) In the region of low electron density ka decreases from the solution of the empty waveguide with increasing density towards the cut-off ($ka = 0$). This propagation mode is called “waveguide-mode” (Trivelpiece⁷) for the plasma only modifies the wave propagation in the waveguide.
- ii) When the electron density increases further, wave propagation is possible at densities above the dipole resonance at $n/n_c = 2$ ($\omega^2 = \omega_p^2/2$); in this region the plasma causes the wave propagation – also in case frequencies just below the cut-off frequency of the empty waveguide. Therefore this mode is called “plasmaguide-mode” (Trivelpiece⁷). For very high electron densities the plasma behaves like a metallic conductor and so the plasmaguide mode passes over into the coaxial H_{11} -waveguide mode.

The electrostatic theory coincides with these results only around the dipole resonance. The electromagnetic waves as well in the empty waveguide as in the coaxial waveguide cannot be described electrostatically.

The dipole resonance of a plasma column in a homogeneous electric field occurs when $\epsilon_{\text{inside}} = -\epsilon_{\text{outside}}$ (Herlofson¹⁴). For that reason one expects a shift of the resonance to higher electron densities when a discharge tube is taken into account (cf. Section 3.3.).

In Figure 3 the field patterns of the electric field are given in the cross section of the plasma-waveguide-system for $r_p/a = 0.3$. E_φ is constant at the boundary.

Since $n/n_c < 2$ for the waveguide mode one gets $|\epsilon_p| < 1$; so the radial electric field strength will be greater in the plasma than outside in the waveguide. In case of the plasmaguide mode, $|\epsilon_p| \geq 1$ and the radial component of the electric field strength in the

plasma will be smaller than outside in the waveguide.

3.2. Homogeneous Plasma Column in a Circular Waveguide – with Collisions ($\nu \neq 0$)

As quoted before, collisions of the electrons may be taken into account by using a complex dielectric constant (34). The normalized wave vector $k \cdot a$ is then complex too: the real part describes the wavelength and the imaginary part the decrease of the wave. Using the notation (7) and (8) a negative value of $\text{Im}(ka)$ describes a damped wave propagating in z -direction.

The dispersion relation for low ν/ω is shown in Figure 4. In the waveguide mode the absorption increases with increasing electron density until $-\text{Im}(ka) > \text{Re}(ka)$. In the plasmaguide mode the absorption increases sharply with decreasing electron density near the dipole resonance. For further decreasing density the mode changes – in this special case – to the E_{12} -mode of the empty waveguide, which does not propagate.

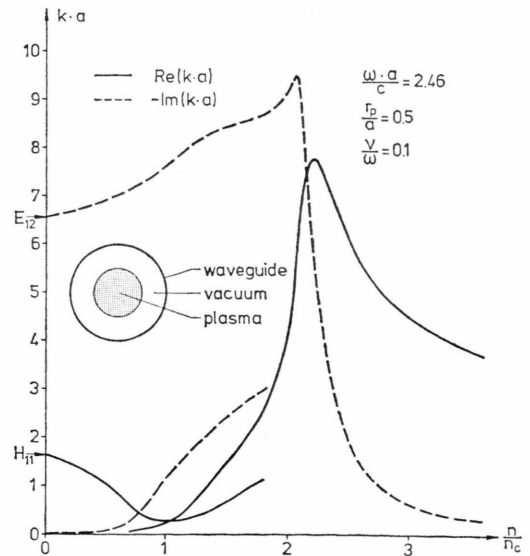


Fig. 4. Dispersion relation in case of low collision frequency.

The dispersion relation for higher values of ν/ω is plotted in Figure 5. The resonance phenomenon shows up strongly for low ν/ω and is weakened for high ν/ω . In case of $\nu/\omega = 1.0$ there is a damped coaxial mode, where $\text{Re}(ka) > -\text{Im}(ka)$ is valid for all densities.

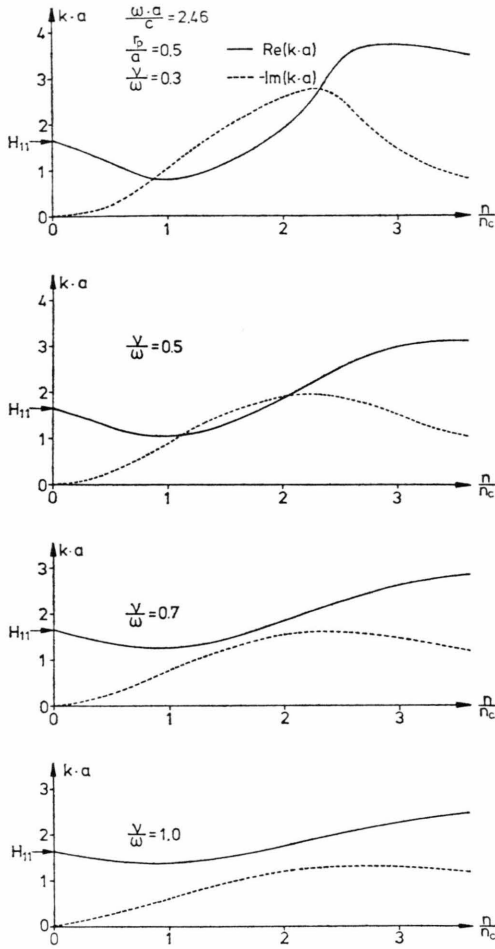


Fig. 5. Modification of the dispersion relation with increasing collision frequency.

The calculated absorption is extremely high; $-\text{Im}(ka) = 1$ implies, that the amplitude of the wave is lowered by a factor $1/e$ along a distance of the waveguide's radius a .

A survey of the absorption in a plasma-waveguide-system as a function of the parameters n/n_c and ν/ω is given in Fig. 6 for two values of r_p/a .

The absorption is very strong for low ν/ω near the dipole resonance. For $r_p/a = 0.5$ the region of high absorption is wider than for the lower value $r_p/a = 0.3$. For $\nu/\omega = 0$ the region $-\text{Im}(ka) > \text{Re}(ka)$ corresponds to the region between cut-off and dipole resonance (cf. Figure 2).

3.3. The Influence of a Discharge Tube on the Dispersion Relation

Figure 7 shows the displacement of both the cut-

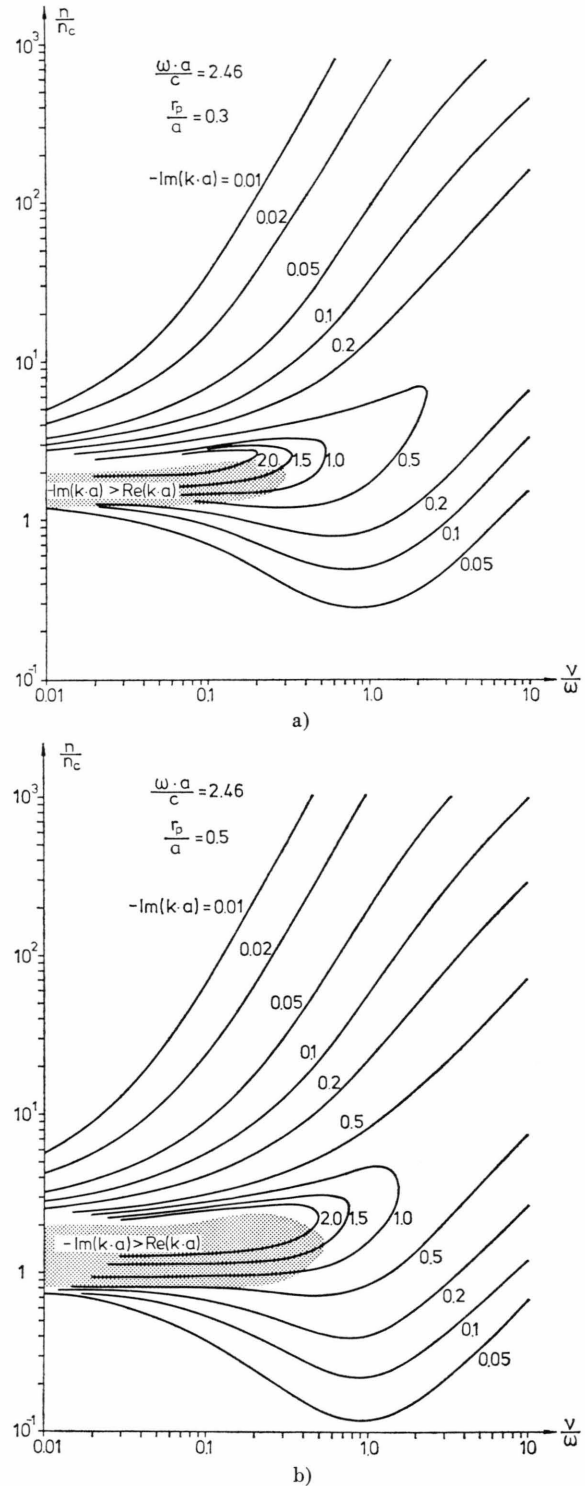


Fig. 6 (a, b). Survey of the absorption in a coaxial plasma-waveguide-system as function of n/n_c and ν/ω
a) $r_p/a = 0.3$ b) $r_p/a = 0.5$.

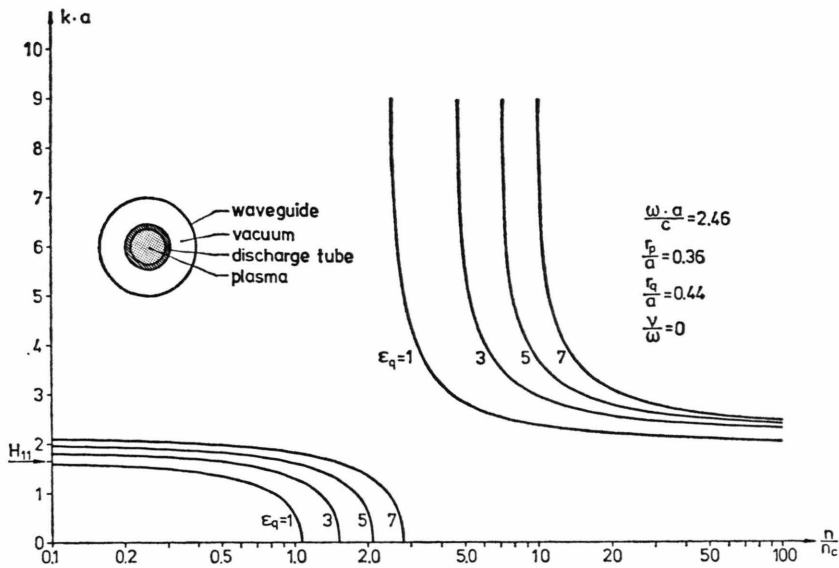


Fig. 7. Shift of cut-off and of dipole resonance caused by a discharge tube.

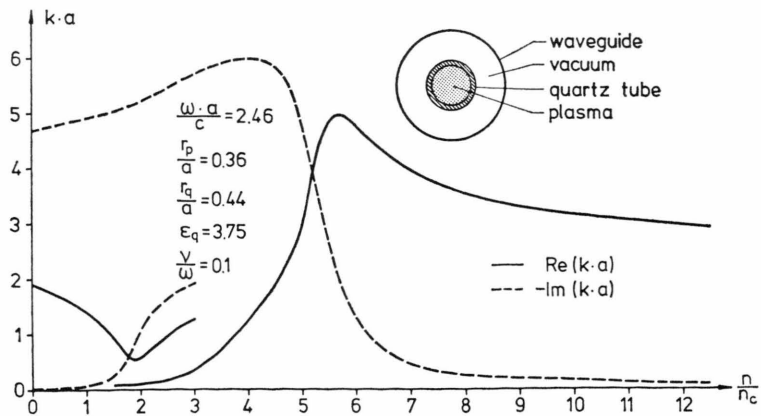
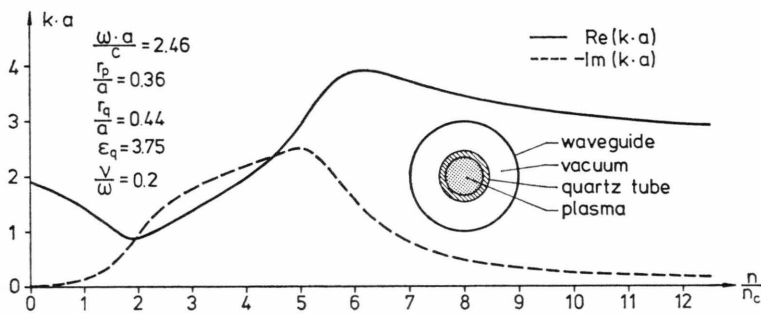


Fig. 8. Dispersion relation for a homogeneous plasma column enclosed by a quartz tube ($\epsilon_q = 3.75$) for $\nu \neq 0$.

off and the dipole resonance to higher electron densities caused by a discharge tube. The displacement increases with increasing values of the relative dielectric constant of the discharge tube ϵ_q .

For the case of damping, Fig. 8 shows examples of the dispersion relation in presence of a quartz tube. The shift of the resonance to higher electron densities is obvious (cf. Figure 4). The resonance

seems less pronounced as compared to the case without discharge tube.

3.4. Plasma Column with Radial Gradients of Electron Density in a Circular Waveguide

As discussed in Sect. 2, a radial gradient of electron density is approximated by a step profile of electron density. In order to avoid numerical problems, the sequence of the layers has to be arranged in a special manner: the continuity of D_r causes an increase of E_r near $n/n_c = 1$. Therefore around this region the layers should be kept thin and one interface should be placed at the radius corresponding to $n/n_c = 1$; in case of equidistant interfaces "artificial" resonances show up. An example for the distribution of 20 layers is shown in Figure 9: the radius where $n/n_c = 1$ is valid, is given for a Bessel profile approximately by

$$r_c = r_p \left(1 - \frac{1}{(n/n_c)(r=0) \cdot 1.25} \right). \quad (35)$$

A third of the layers is placed between r_c and r_p , a third is distributed over the neighbouring equidistant range. The remaining number of layers is distributed along the inner part of the plasma column.

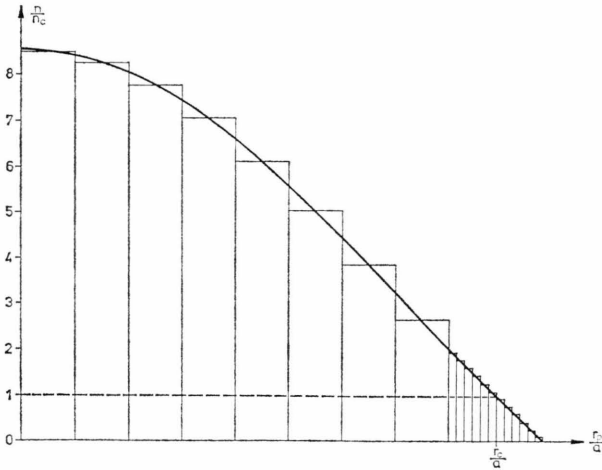


Fig. 9. Distribution of 20 layers over the radius of a plasma column with a radial Bessel profile for a specific value of n/n_c .

In view of experimental results¹⁵ several relative profiles of electron density are considered (Fig. 10): the density gradient of the profile at $r = r_p$ is measured in multiples of the Bessel function J_0 ; the profiles are cut at the same height as J_0 .

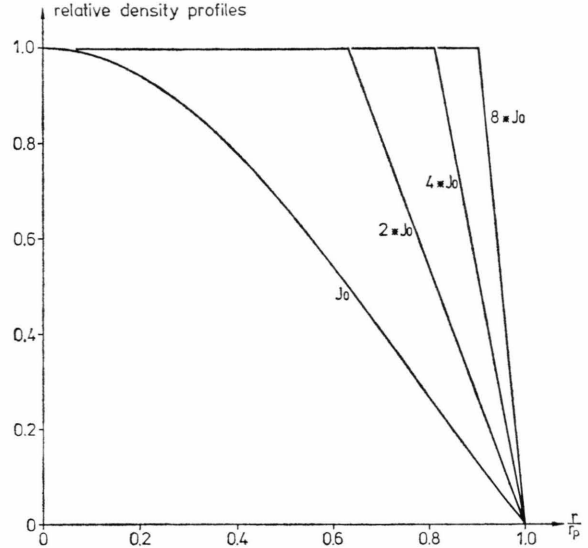


Fig. 10. Definition of relative density profiles $n * J_0$.

The distribution of the layers is chosen analogically to Figure 9. The results of the calculation are shown in Fig. 11; as one would expect the absorption decreases — for the same electron density at the axis of the plasma column — with increasing density gradient, for the electromagnetic wave penetrates the plasma column less and less. For very high gradients of electron density at the edge of the plasma column, the plasma column behaves like a homogeneous plasma column.

Near the dipole resonance the dispersion relations differ not only in the imaginary part but also in the real part of $k \cdot a$, so that one may get information as well on the electron density as on the density profile from measured values of $\text{Re}(k a)$ and $\text{Im}(k a)$.

Figure 13 demonstrates to what extent the absorption takes place in the outer part of the plasma column; the absorption is plotted for the various density profiles as function of the normalized density gradient $\partial(n/n_c)/\partial r \cdot r_p$. For high electron densities the clipping of the profile to 25% of the height (cf. Fig. 12) does not alter the absorption seriously; this implies that the absorption occurs in the outer part of the plasma column. From Fig. 13 follows that the absorption in the plasma-waveguide-system for high electron densities is a function of the radial gradient of the electron density if the extension of the gradient covers more than approximately 20...30% of the radius of the plasma column; the shape of the density profile in the inner

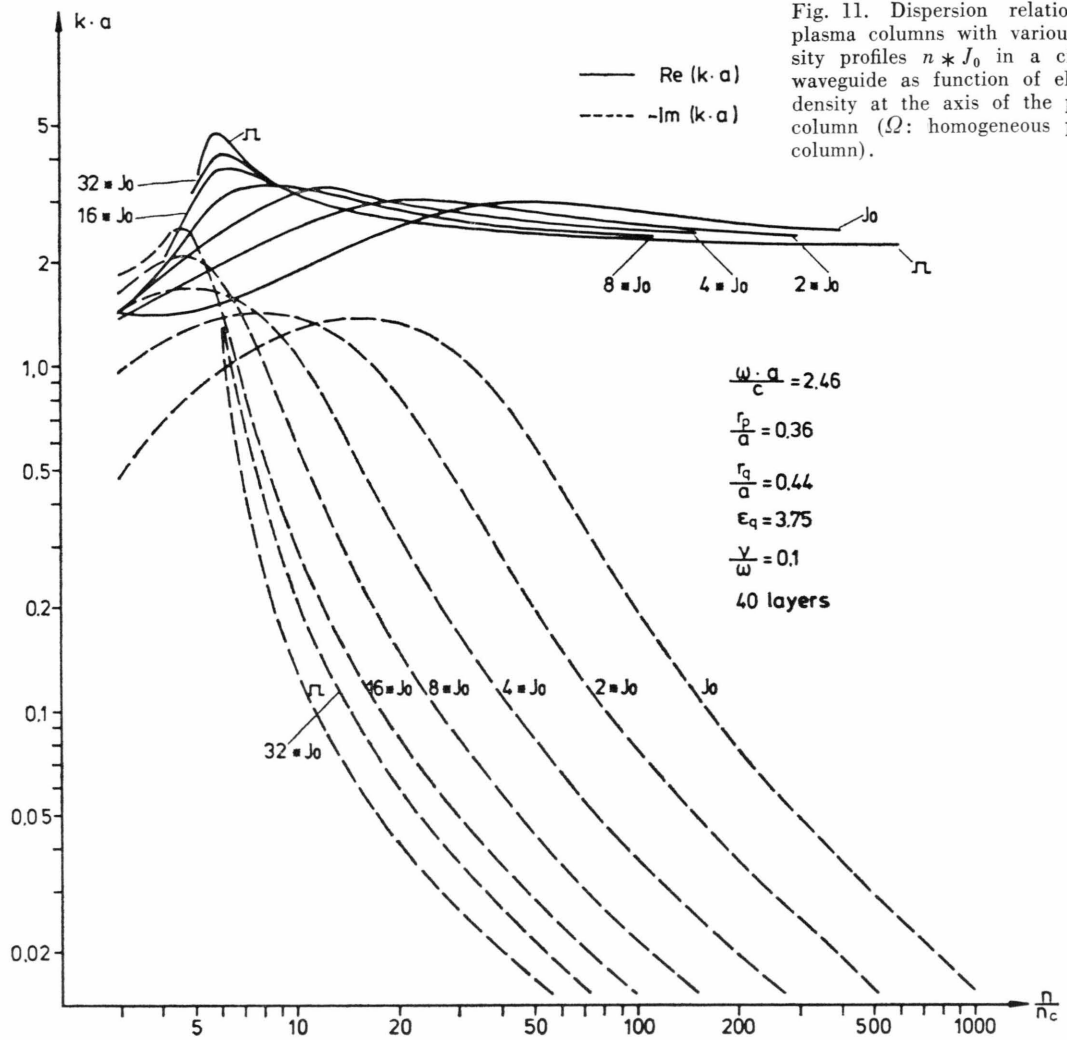


Fig. 11. Dispersion relation for plasma columns with various density profiles $n * J_0$ in a circular waveguide as function of electron density at the axis of the plasma column (Ω : homogeneous plasma column).

part of the plasma does not affect the absorption considerably.

4. Conclusions

Propagation and absorption of electromagnetic waves have been studied in a coaxial plasma-waveguide-system. Due to the dipole resonance of the plasma column very high absorption is obtained also for low values of ν/ω . The absorption is greatly influenced by radial density gradients. In case of high electron density the gradient of electron density at the edge of the plasma column determines the absorption for a given collision frequency ν . This high absorption can be used to generate overdense plasmas¹⁵.

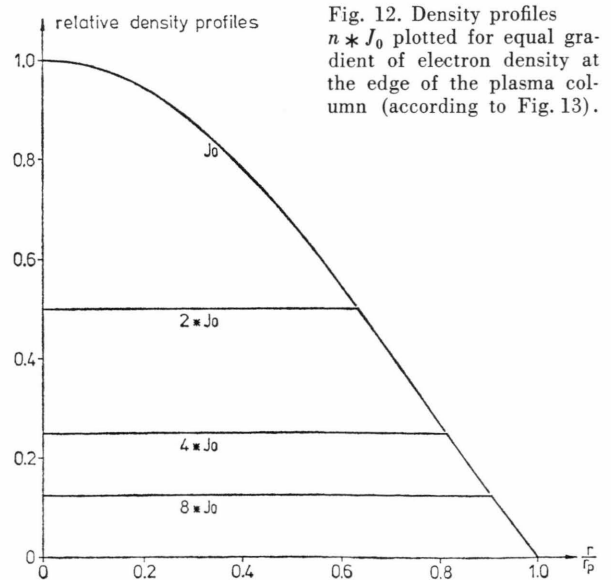


Fig. 12. Density profiles $n * J_0$ plotted for equal gradient of electron density at the edge of the plasma column (according to Fig. 13).

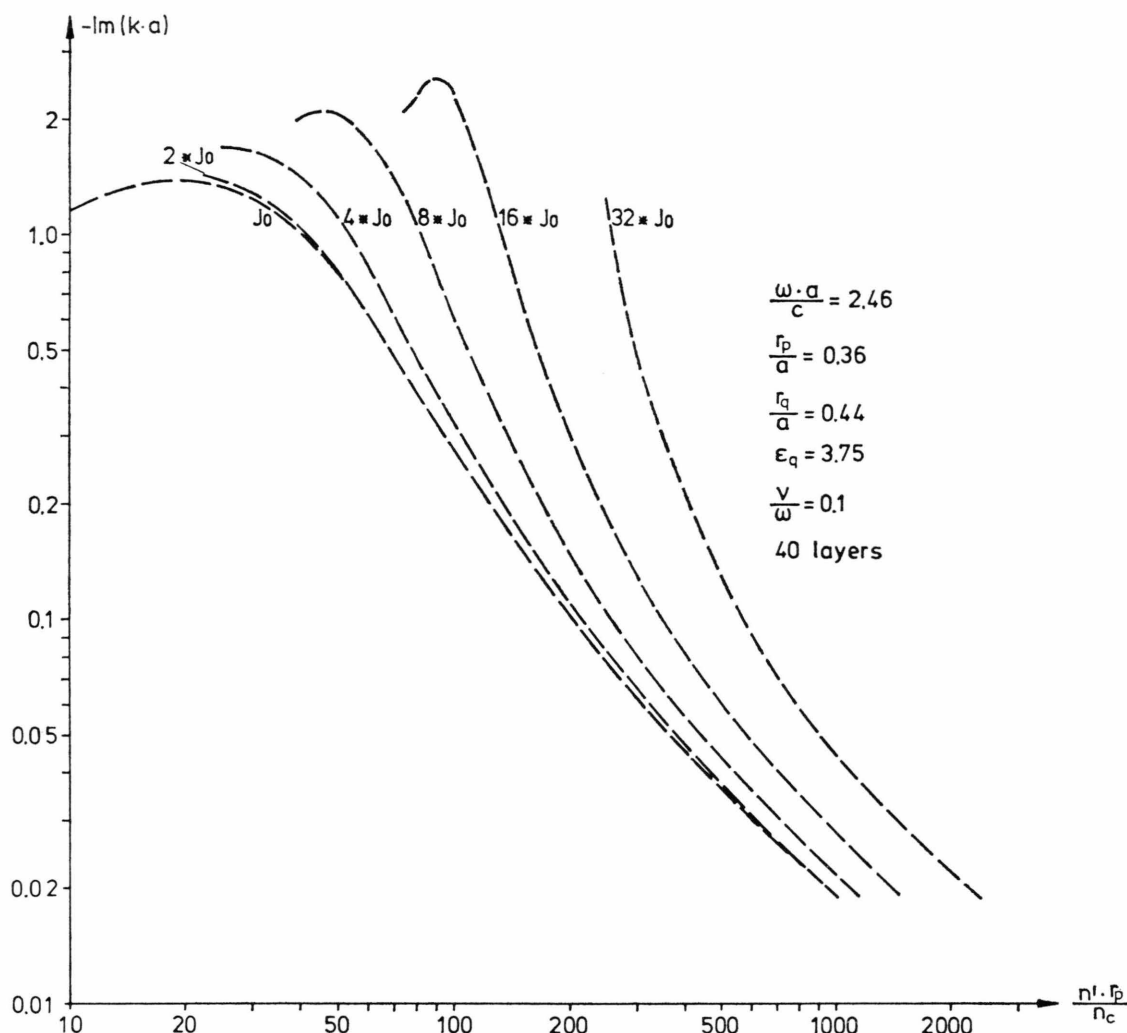


Fig. 13. Absorption in a plasma-waveguide-system als function of the normalized density gradient $\frac{\partial (n/n_c)}{\partial r} \cdot r_p$ at the edge of the plasma.

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These investigations are part of joint efforts within the Sonderforschungsbereich 162 "Plasma-physik Bochum/Jülich". The author wishes to thank Prof. Dr. H. Schlüter and Dr. H. Beerwald for the support of the work and Mr. K. Brinkhoff for his assistance in plotting the diagrams.

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